Test 3A - MTH 1410

Dr. Graham-Squire, Spring 2014

8:57
9:15
→ 24 min.

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 6 questions of the test.
5. You will be able to come back to the calculator portion of the test, but you cannot come back to the No Calculator portion after you turn it in.
6. If you need it, the quadratic formula is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
8. Make sure you sign the pledge.
9. Number of questions = 9. Total Points = 60.

1. (5 points) Sketch a curve that satisfies the following:

   $f(0) = 0$ $\lim_{x\to\infty} f(x) = 2$ $\lim_{x\to(-\infty)} f(x) = 2$

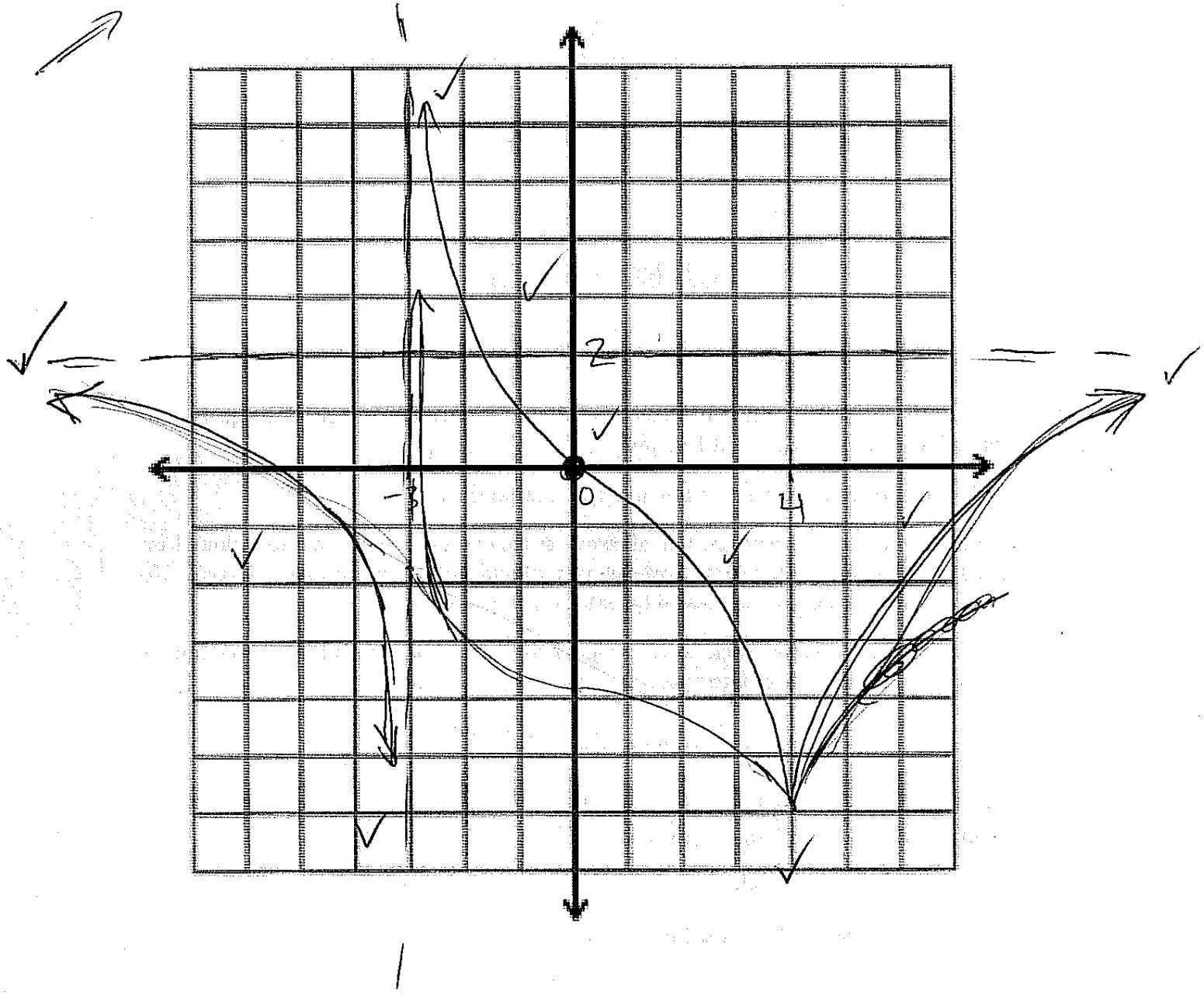
   $f'(x) > 0$ on the interval $(4, \infty)$.

   $f'(x) < 0$ on the intervals $(-\infty, -3)$ and $(-3, 4)$.

   $f''(x) > 0$ on the interval $(-3, 0)$.

   $f''(x) < 0$ on the intervals $(-\infty, -3)$, $(0, 4)$, and $(4, \infty)$.

   There is one discontinuity. at $x = -3$



2. (4 points) Calculate the limit:

$$\lim_{x\to 0^+} x^{(2/\ln x)}$$

$0^{(2/-\infty)} = 0^0$ indeterminate!

$y = x^{2/\ln x}$ ✓

$\ln y = \ln x^{(2/\ln x)}$ ✓

$\ln y = \frac{2}{(\ln x)} \cdot (\ln x)$ ✓ ✓

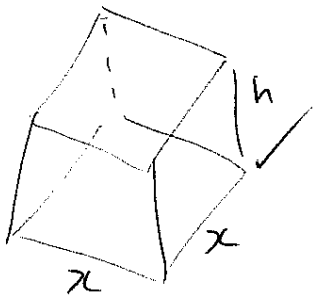
$\ln y = 2$ ✓

✓ $\lim_{x\to 0^+} \ln y = 2$

$\Rightarrow \lim_{x\to 0^+} y = \boxed{e^2}$ ✓

3. (8 points) A box (that is, a rectangular prism) with a square base is to be made with a volume of 30 ft^3. The material for the sides costs \$10 per square foot, the material for the bottom costs \$3 per square foot, and the material for the top costs \$15 per square foot. Use calculus to find the dimensions of the cheapest possible box.

Round to nearest 0.01.



$V = x^2h$

Cost $= 10(4xh) + 3(x^2) + 15(x^2)$

$C = 40xh + 18x^2$ ✓

✓ $30 = x^2h \Rightarrow h = \frac{30}{x^2}$

$\Rightarrow C(x) = 40x\left(\frac{30}{x^2}\right) + 18x^2$ ✓

$C'(x) = \frac{1200}{x} + 18x^2$

$C'(x) = \frac{-1200}{x^2} + 36x$ ✓

$0 = \frac{-1200 + 36x^3}{x^2}$ ✓

$\Rightarrow -1200 + 36x^3 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{\frac{1200}{36}}$ ✓

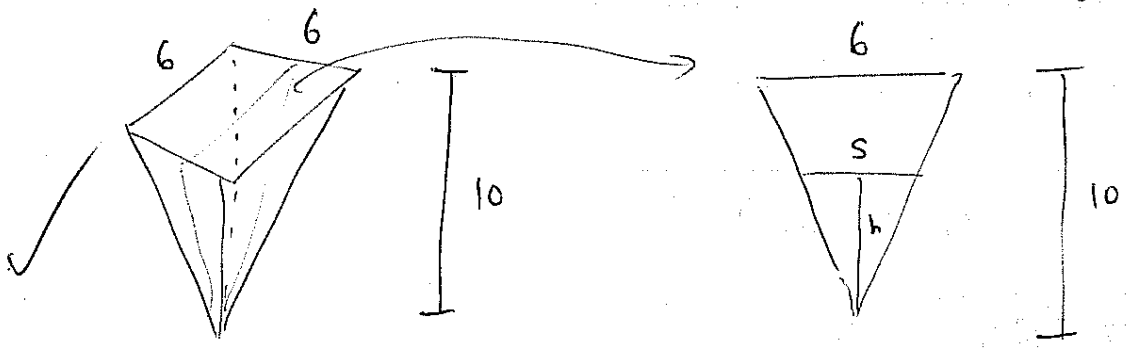
$x = 3.22$

$\Rightarrow h = 30/(3.22)^2 = 2.90$ ✓

total

−1.5 for surface area

4. (8 points) A water cistern is in the shape of an inverted pyramid (thus the pyramid's base, which is a square, is at the top, and the point of the pyramid is at the bottom of the cistern). The cistern is full but is draining from the bottom at a rate of 4 ft^3 per minute. If the sides of the base of the cistern have length 6 feet, and the cistern has a height of 10 feet, how fast is the height of the water decreasing when the water level reaches 3 feet? Note: The volume of a square pyramid is given by $V = \frac{1}{3}s^2h$.



Round to nearest 0.01.

✓ Know: $\frac{dV}{dt} = -4$

Want $\left.\frac{dh}{dt}\right|_{h=3}$

$\frac{s}{h} = \frac{6}{10}$ ✓

$s = \frac{3}{5}h$

$V = \frac{1}{3}s^2h$

$V = \frac{1}{3}\left(\frac{3}{5}h\right)^2 \cdot h$ ✓

$\frac{d}{dt}\left(V = \frac{3}{25}h^3\right)$ ✓

$\frac{dV}{dt} = \frac{3}{25} \cdot 3h^2 \frac{dh}{dt}$ ✓

$-4 = \frac{9}{25}(3^2)\frac{dh}{dt}$ ✓

$\frac{-4(25)}{81} = \frac{dh}{dt}$

$\Rightarrow \frac{-100}{81} \approx -1.23 = \frac{dh}{dt}$ ✓

height is decreasing at ≈1.23 ft/sec

5. (8 points) The stopping distance for a car is given by

$$f(x) = 2.5x + 0.5x^2$$

where x is the speed of the vehicle and d is the distance (in feet) it takes for the vehicle to stop once the brakes are applied. Use linearization or differentials to approximate the change in the stopping distance when speed increases from $x = 27.5$ to $x = 28$.

$dy = f'(s)\,dx$ ✓✓✓

$s = 27.5$ ✓

$dx = 0.5$ ✓

$f'(x) = 2.5 + x$ ✓

$dy = (2.5 + 27.5)(0.5)$ ✓

$dy = 15$ feet ✓

or

$L(x) = f(p) + f'(p)(x - p)$ ✓✓✓

$p = 27.5$ ✓

$f'(x) = 2.5 + x$ ✓

$f'(p) = 30$ ✓

$f(p) = 2.5(27.5) + 0.5(27.5)^2$

$f(27.5) = 446.875$

$L(x) = 446.875 + 30(x - 27.5)$ ✓

$L(28) = 446.875 + 30(0.5)$

$= 446.875 + 15$ ✓

15 → change

6. (8 points) Use calculus to find the intervals of concavity and inflections points (if any exist) for the function

$$f(x) = e^{x-x^2}$$

Make sure to show your work to receive full points!

Round to nearest 0.01

$f'(x) = e^{x-x^2}(1-2x)$ ✓

$f''(x) = e^{x-x^2}(1-2x)(1-2x) + (-2)e^{x-x^2}$ ✓✓

$= (e^{x-x^2})\left((1-2x)(1-2x) - 2\right)$

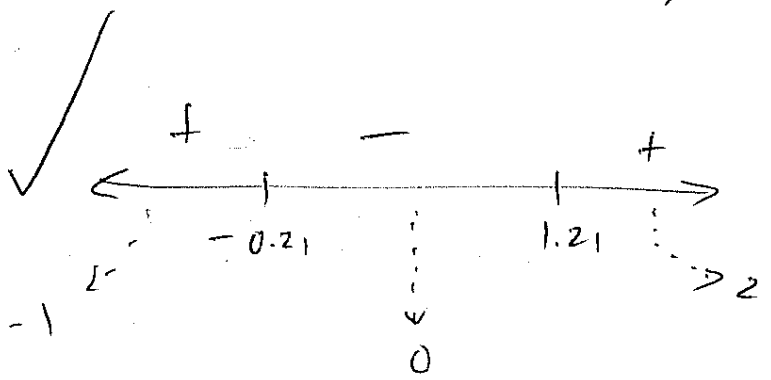
✓ $0 = e^{x-x^2}(1-4x+4x^2-2)$ $\quad f''(x) = e^{x-x^2}(4x^2-4x-1)$

$0 = e^{x-x^2}$ never

$0 = 4x^2-4x-1 \Rightarrow \frac{4 \pm \sqrt{16-4(4)(-1)}}{8} = x$ ✓

✓

$x = \frac{4 \pm \sqrt{32}}{8} \Rightarrow x = 1.21$ or $x = -0.21$



$f''(-1) = +$

$f''(0) = -$

$f''(2) = +$

- conc up on $(-\infty, -0.21)$ and $(1.21, \infty)$
- conc. down on $(-0.21, 1.21)$

inf. points $(-0.21, 0.78)$ $(1.21, 0.78)$

Test A **No Calculator** Name: Key

7. (6 points) Calculate the limits: ✓

(a) $\lim_{x\to\frac{\pi}{2}^+} \dfrac{\cos x - 1}{\sin x}$ $\rightsquigarrow$ $\dfrac{0-1}{1} = \dfrac{-1}{1} = \boxed{-1}$ ✓

$\boxed{-1}$

(b) $\lim_{x\to 1} \dfrac{x^2 - x - \ln x}{\cos(\pi x) + 1}$ $\rightsquigarrow$ $\dfrac{1-1-0}{-1+1} = \dfrac{0}{0}$ ✓ (−½ if they skip and do L'H)

✓ $\overset{L}{=} \lim_{x\to 1} \dfrac{2x - 1 - \frac{1}{x}}{-\pi \sin(\pi x)}$ $\rightsquigarrow$ $\dfrac{2-1-1}{-\pi \cdot 0} = \dfrac{0}{0}$

✓ $\overset{L}{=} \lim_{x\to 1} \dfrac{2 + \frac{1}{x^2}}{-\pi^2 \cos(\pi x)} = \dfrac{3}{-\pi^2(\cos\pi)} = \dfrac{3}{\pi^2(-1)} = \boxed{\dfrac{3}{\pi^2}}$ ✓

up to two calculation errors $\Rightarrow -0.5$

more $\Rightarrow -1$

8. (8 points)

(a) Let $f'(x) = \sin x + 2e^x + 2$. If $f(0) = 5$, find the expression for $f(x)$.

$$f(x) = \int (\sin x + 2e^x + 2)\, dx \quad \checkmark$$

$$f(x) = -\cos x + 2e^x + 2x + C \quad \checkmark$$

$$5 = f(0) = -\cos(0) + 2e^0 + 2(0) + C \quad \checkmark$$

$$5 = -1 + 2 + C$$

$$4 = C \quad \checkmark$$

$$\Rightarrow \boxed{f(x) = -\cos x + 2e^x + 2x + 4}$$

(b) Find the most general antiderivative:

$$\int \left(\frac{x + x^4 + x^8}{x^5}\right) dx$$

← Hint: Try to simplify the expression first.

$$= \int \left(\frac{x}{x^5} + \frac{x^4}{x^5} + \frac{x^8}{x^5}\right) dx \qquad 0.5$$

$$= \int \left(x^{-4} + \frac{1}{x} + x^3\right) dx \qquad 0.5$$

$$= \boxed{\frac{x^{-3}}{-3} + \ln|x| + \frac{x^4}{4} + C}$$

✓ 0.5 ✓ 0.5

9. (5 points) Use calculus to find the absolute maximum and absolute minimum of

$$f(x) = \frac{3x-4}{x^2+1}$$

Move to calc. section.

on the interval [-2,2]. Make sure to show your work!

$$f'(x) = \frac{3(x^2+1) - (3x-4)(2x)}{(x^2+1)^2} \quad \checkmark\checkmark$$

$$\Rightarrow 0 = 3x^2 + 3 - 6x^2 + 8x \quad \checkmark$$

$$0 = -3x^2 + 8x + 3$$

$$0 = 3x^2 - 8x - 3$$

$$0 = (3x+1)(x-3) \quad \checkmark$$

$\checkmark$ $3x+1=0 \Rightarrow x = \frac{-1}{3}$ $\quad x-3=0 \Rightarrow x=3$

Not in domain!

Check $\checkmark$ $f\left(\frac{-1}{3}\right) = \frac{-1-4}{\frac{1}{9}+1} = \frac{-5}{\left(\frac{10}{9}\right)} = \frac{-45}{10} = \boxed{-4.5}$ $\checkmark$ min

$\checkmark$ $f(-2) = \frac{-6-4}{4+1} = \frac{-10}{5} = -2$

$\checkmark$ $f(2) = \frac{6-4}{5} = \frac{2}{5} = \boxed{0.4}$ ← max $\checkmark$

Extra Credit(up to 2 points) Choose 0.5 or 2 points for extra credit. If you choose 0.5, you are guaranteed to get the extra half-point. If you put 2 points, and more than half of the class also chooses 2 points, then you get nothing. If less than half the class chooses 2, you get 2 points.

0.5	2	No choice
~~IIII~~ II	II	I